Chapter 1

Assignment 1 Solutions

1.2. a. Calculate the data rate required to transmit a 20,000 Hz audio signal at a sampling rate of four times the Nyquist rate with a digitization of 8 bits per sample. Calculate the channel bandwidth.

b. . . . for a 5 MHz signal?

Solution: a. The Nyquist sampling rate is 2x the highest frequency (assumed equal to the bandwidth); so, the sampling rate S is

$$S = 4 \times 2\,\mathrm{BW} = 8\,\mathrm{BW}\,. \tag{1.1a}$$

The bit rate, then, is

$$\begin{aligned} B_R &= S \times N = 8\,\mathrm{BW} \times 8 = 64\,\mathrm{BW} = (64)(20 \times 10^3) \\ &= 1.280 \times 10^6 = 1.28\ \mathrm{Mb} \cdot \mathrm{s}^{-1}\,. \end{aligned} \tag{1.1b}$$

The receiver and transmitter channel bandwidth required is about one-half of the bit rate, so

$$B \approx \frac{B_R}{2} = 0.64\ \mathrm{MHz} = 640\ \mathrm{kHz}\,. \tag{1.2}$$

b. For a 5 MHz signal,

$$B_R = 64\,\mathrm{BW} = (64)(5 \times 10^6) = 3.20 \times 10^7 = 320\ \mathrm{Mb} \cdot \mathrm{s}^{-1}\,. \tag{1.3}$$

The bandwidth required is about one-half of the bit rate, so

$$\mathrm{BW} \approx \frac{B_R}{2} = 160\ \mathrm{MHz}\,. \tag{1.4}$$

1.3 Calculate the data rate required to transmit a high-definition television (HDTV) signal if the image is 1000×1000 pixels, each pixel is tricolor with 12 bits of resolution per color, and the frame rate is 70 frames per second.

Solution: The bit rate would be

$$B_R = \underbrace{(10^3)(10^3)}_{\text{pixels/frame}} \underbrace{(3)(12)}_{\text{bits/pixel}} \underbrace{(70)}_{\text{frame/s}} = 2.52 \times 10^9 \text{ b/s} = 2.52 \text{ Gb/s}\,. \tag{1.5}$$

2.3. Light traveling in air strikes a glass plate with an angle of incidence of 57 degrees.

a. If the reflected and refracted beams make an angle of 90 degrees with each other, calculate the refractive index of the glass.

b. What is the critical angle for this material if the light travels from glass into air?

Solution: See Fig. 1.1 for the geometry of this problem.

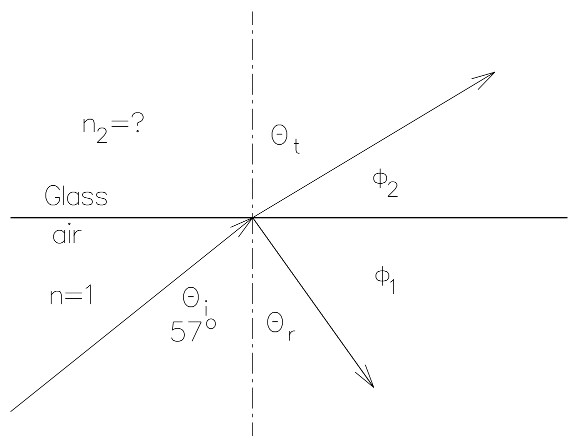

Figure 1.1: Geometry for Problem 2.3.

a. We have $\theta_i = 57°$ and that $\phi_1 + \phi_2 = 90°$. Since

$$\theta_R = \theta_i = 57°\,, \tag{1.6}$$

then,

$$\phi_1 = 90° - \theta_R = 90 - 57 = 33° \tag{1.7a}$$

and

$$\phi_2 = 90° - \phi_1 = 90 - 33 = 57°\,. \tag{1.7b}$$

But also

$$\theta_t = 90 - \phi_2 = 90 - 57 = 33^\circ\,. \tag{1.8}$$

By Snell's law

$$n_1 \sin\theta_i = n_2 \sin\theta_t \tag{1.9a}$$

so

$$n_2 = n_1 \left(\frac{\sin\theta_i}{\sin\theta_t}\right) = 1.00 \left(\frac{\sin 57^\circ}{\sin 33^\circ}\right) = 1.54\,. \tag{1.9b}$$

b. We find the critical angle from

$$\theta_c = \sin^{-1}\left(\frac{n_1}{n_2}\right) = \sin^{-1}\left(\frac{1.0}{1.54}\right) = 40.5^\circ\,. \tag{1.10}$$

2.4. A point source of light is located 1 m below a water-air interface. Find the radius of the light circle seen by an observer positioned over the source. The refractive index of water is 1.333.

Solution: See Fig. 1.2 for the geometry of the problem. We know that the critical angle is given by

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.0}{1.33}\right) = 48.8^\circ\,. \tag{1.11}$$

From trigonometry we have

$$R = d\tan\theta_c = (1.0)\tan(48.8^\circ) = 1.14 \text{ m}\,. \tag{1.12}$$

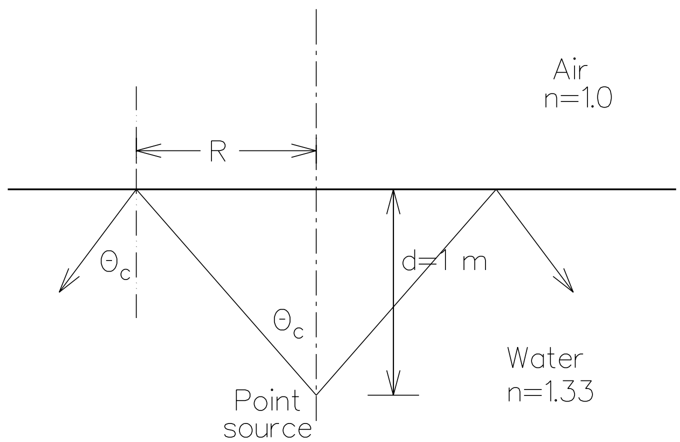

Figure 1.2: Geometry for Problem 2.4.